The surface $OA'A$ is the total conical surface less the ungula, that is,

$$\frac{\pi}{2}\sin\alpha[2K'^2 - 2K'^2 + (K+K')\sqrt{KK'}] = \frac{\pi}{2}(K+K')(KK')^{\frac{1}{2}}\sin\alpha.$$

The problem rests upon the tedious integration of the form,

$$\int_r^R 2x\cos^{-1}\left[\frac{2Rr-(R+r)x}{(R-r)x}\right]dx.$$

Also solved by A. M. Harding and P. Penalver.

A solution of 334 was received from Elmer Schuyler, after the the forms were sent to the printer.

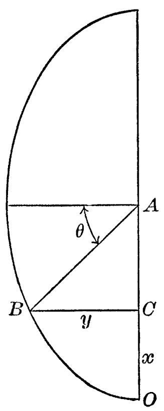
MECHANICS.

270. Proposed by W. J. GREENSTREET, Burghfield, England.

A cycloid has its base vertical. Find the line of quickest descent from the middle point of the base, and its approximate inclination to the horizon.

Solution by J. Scheffer, Hagerstown, Md.

Let AB be any straight line drawn from the middle point A of base of the cycloid, and let θ be the angle that it makes with the horizon. The time of descent from A to B is $t=\sqrt{\frac{2}{g}\cdot\frac{s}{\sin\theta}}$, AB being $=s$. But $s^2=(r\pi-x)^2+y^2$, x and y being subject to the equations of the cycloid, $x=r(\phi-\sin\phi)$, $y=r(1-\cos\phi)$. Substituting, we have $s^2=r^2[(\pi-\phi)+\sin\phi]^2+(1-\cos\phi)^2$. We have to get the minimum of $s/\sin\theta$, which reduces to



$$\frac{(\psi+\sin\psi)^2+(1+\cos\psi)^2}{\psi+\sin\psi},$$

after putting $\pi-\phi=\psi$ and omitting the constant r^2.

Differentiating and setting the differential coefficient equal to zero, we get, after some easy reductions, the transcendental equation $\psi^2-2\cos\psi-2=0$. Whence $\psi=\pm 2\cos\psi/2$, and using the positive sign, we find $\psi=85°$, nearly. Then

$$\tan\theta=\frac{r\pi-x}{y}=\frac{\psi+\sin\psi}{1+\cos\psi}.$$

Hence, finally, $\theta=53°\ 39'$, nearly.

Note. It is assumed in the above solution that the line sought is the *straight line* of quickest descent. Otherwise the problem becomes much more difficult, requiring for its solution the calculus of variations. The Editors.

NUMBER THEORY.

A solution of 188 was received from Elmer Schuyler after the December issue had been made up.